



Examen Math2- Mai 2016. Durée : 90 Minutes

Exercice 01 (08pts) : Soit la matrice :

$$A = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 2 & 1 \\ 2 & 1 & -1 \end{pmatrix}$$

1. Montrer que $\det A = \det \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 3 & -9 \end{pmatrix}$

برهن ان

2. Calculer A^{-1}

احسب

3. Résoudre le système d'équations linéaires suivant

حل النظام الاتي

a) En utilisant A^{-1}

باستعمال A^{-1}

b) En utilisant la méthode de Cramer

باستعمال طريقة كرامر

$$\begin{cases} x - y + z = 1 \\ -x + 2y + z = 2 \\ 2x + y - z = 3 \end{cases}$$

Exercice 02 (06pts) : On donne :

$$A = \begin{pmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{pmatrix} \text{ et } B = \begin{pmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{pmatrix}$$

نعطي

Montrer que l'on a :

1. $AB = BA = 0$

برهن أن

2. $A^2 - B^2 = (A - B)(A + B)$

3. $(A + B)^2 = A^2 + B^2$

Exercice 03 (06pts) :

Calculer :

احسب:

$$\int_0^1 \frac{x^2 + x + 1}{(x+1)(x^2+1)} dx$$

On pourra écrire $\frac{x^2+x+1}{(x+1)(x^2+1)} = \frac{a}{x+1} + \frac{bx+c}{x^2+1}$

يمكن كتابة:

Bon Courage

Corrigé type de l'examen Math II
Mai 2016

exo 1

1- $\det A = -9 = \begin{vmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 3 & -9 \end{vmatrix} \dots \dots (1 \text{ pt})$

2- $A^{-1} = \frac{1}{\det A} \cdot {}^t(\text{com} A) \Rightarrow$

${}^t \text{Com} A = \begin{pmatrix} -3 & 0 & -3 \\ 1 & -3 & -2 \\ -5 & -3 & 1 \end{pmatrix} \dots \dots (1 \text{ pt})$

$\Rightarrow A^{-1} = -\frac{1}{9} \begin{pmatrix} -3 & 0 & -3 \\ 1 & -3 & -2 \\ -5 & -3 & 1 \end{pmatrix} \dots \dots (1 \text{ pt})$

3- a) A^{-1} ? $A X = b \Rightarrow X = A^{-1} b \dots \dots (2 \text{ pt})$

$\Rightarrow X = -\frac{1}{9} \begin{pmatrix} -12 \\ -11 \\ -8 \end{pmatrix} \Rightarrow \begin{cases} x = 4/3 \\ y = 11/9 \\ z = 8/9 \end{cases}$

b) Cramer

$x = \frac{\begin{vmatrix} 1 & -1 & 1 \\ 2 & 2 & 1 \\ 3 & 1 & -1 \end{vmatrix}}{-9} = \frac{4}{3}$

$y = \frac{\begin{vmatrix} 1 & 1 & 1 \\ -1 & 2 & 1 \\ 2 & 3 & -1 \end{vmatrix}}{-9} = \frac{11}{9}$

$z = \frac{\begin{vmatrix} 1 & -1 & 2 \\ 1 & 2 & 2 \\ -2 & 1 & 3 \end{vmatrix}}{-9} = \frac{8}{9} \dots \dots (03 \text{ p})$

ex02:

$$1) \quad A \cdot B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \dots \quad 0,5$$

$$B \cdot A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \dots \quad 0,5$$

$$2) \quad \left. \begin{aligned} A^2 &= A \cdot A = \begin{pmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{pmatrix} \\ B^2 &= B \cdot B = \begin{pmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{pmatrix} \end{aligned} \right\} \quad 0,5$$

$$A^2 - B^2 = \begin{pmatrix} 3 & -6 & -10 \\ -2 & 7 & 10 \\ 2 & -6 & -9 \end{pmatrix} \quad 0,5$$

$$\left. \begin{aligned} A - B &= \begin{pmatrix} 3 & -6 & -10 \\ -2 & 7 & 10 \\ 2 & -6 & -9 \end{pmatrix} \\ A + B &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned} \right\} \quad 0,5$$

$$\Rightarrow (A - B) \cdot (A + B) = \begin{pmatrix} 3 & -6 & -10 \\ -2 & 7 & 10 \\ 2 & -6 & -9 \end{pmatrix} \quad 0,5$$

$$3) \quad (A + B)^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad 0,5$$

$$A^2 + B^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad 0,5$$

ex03

$$\frac{a}{x+1} + \frac{bx+c}{x^2+1} = \frac{(a+b)x^2 + (b+c)x + a+c}{(x+1)(x^2+1)} \quad \dots \textcircled{1}$$

$$\Rightarrow \begin{cases} a+b=1 \\ b+c=1 \\ a+c=1 \end{cases} \Rightarrow \begin{cases} a = \frac{1}{2} \\ b = \frac{1}{2} \\ c = \frac{1}{2} \end{cases} \quad \dots \textcircled{1}$$

$$\begin{aligned} \Rightarrow \int_0^1 \frac{x^2+x+1}{(x+1)(x^2+1)} dx &= \frac{1}{2} \int_0^1 \frac{dx}{x+1} + \frac{1}{2} \int_0^1 \frac{x+1}{x^2+1} dx \quad \textcircled{1} \\ &= \frac{1}{2} \int_0^1 \frac{dx}{x+1} + \frac{1}{4} \int_0^1 \frac{2x dx}{x^2+1} + \frac{1}{2} \int_0^1 \frac{dx}{x^2+1} \quad \textcircled{1} \end{aligned}$$

$$\Rightarrow = \frac{1}{2} \ln(x+1) + \frac{1}{4} \ln(x^2+1) + \left[\frac{1}{2} \arctan x \right]_0^1 \quad \dots \textcircled{1}$$

$$= \frac{3}{4} \ln 2 + \frac{\pi}{8} \quad \dots \textcircled{1}$$